Sediment Budgets 1 (Proceedings of symposium S1 held during the Seventh IAHS Scientific Assembly at Foz do Iguaçu, Brazil, April 2005). IAHS Publ. 291, 2005.

Bed load transport described by a one-dimensional gamma functions model

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Abstract Experiments were performed in open channel flows to evaluate one-dimensional stochastic models for sand transport and dispersion. One series was based on the observation of kinematic characteristics of single grains labelled with ^{192}Ir, in a 30-m long, 0.40-m wide channel subjected to liquid discharges of 12 to 33 l s^{-1}. It was shown that the grain rest periods as well as the step lengths could be described by gamma probability density functions. A one-dimensional Gamma Functions Model (1D-GM) was therefore proposed to describe the longitudinal distances travelled by a group of particles as a function of time. The model was fitted to the results of a second series of experiments with grains labelled with ^{198}Au or simulated by crushed glass containing ^{192}Ir. This paper describes those experiments, the 1D-GM properties and its performance in describing Lagrangean behaviour of groups of particles in the laboratory and in natural watercourses.

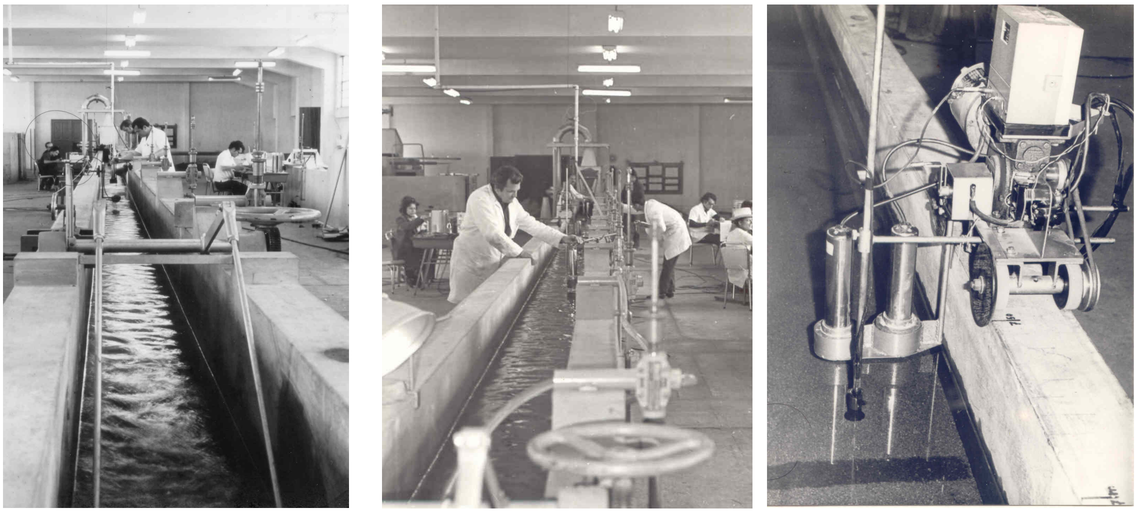
Key words bed load transport; gamma function; model; rivers; sand transport

INTRODUCTION

The longitudinal transport of sediment and contaminant particles in open channel flow as bed load and suspended load can be characterized as a random process, where the elementary events are the trajectories of the single grains. The trajectory of a particle is a chronological series of positive longitudinal displacements in the sense of the resultant flow, interposed by periods of non-displacement in that sense.

In *bed load transport*, the positive displacement takes place by sliding, rolling or small jumps, in such a way that the grain maintains a quasi permanent contact with the bed (Wilson-Jr, 1987). The main null longitudinal displacement periods are the rest periods, when the grain is an integral part of the bottom. In this movement, the most important variable is the mean position of the particles in time and the most convenient description is the Lagrangean description. In *suspended transport*, the resultant transport is always positive, i.e. sediment particles always follow the flow, because displacement is for the most part positive, and both negative and transverse displacements can be ignored. The most important variable to be measured is the mean time of passage of the particles through a section transverse to the flow and the most appropriate description for this movement is the Eulerian description.

To perform these measurements, tracer techniques, involving both radioactive tracers and dyes, have been employed and it has been shown that for the study of sediment movement as a stochastic process such tracer techniques provide the best measurement tool. Using radioactive tracers, and considering the characteristics of these two transport types, a series of studies were performed in laboratory channels and in the field. This paper considers the experiments involving bed load movement.



(a) (b) (c)

Fig. 1 (a) The channel at the Institute of Hydraulic Research of the University of Rio Grande do Sul; (b) single grain manual detection system, and (c) labelled particles injected at the mobile bed and automatic detection system.

DESCRIPTION OF THE EXPERIMENTS ON BED LOAD MOVEMENT

For the bed load studies, two types of laboratory channel experiment were performed at the Institute of Hydraulic Research of the University of Rio Grande do Sul, Brazil: (a) a series of experiments based on observation of the motion characteristics of single radioactive particles labelled with ^{192}Ir; and (b) a series of experiments based on the Lagrangean records of bed load movement involving groups of radioactive particles, in natural and different laboratory channels.

These two types of experiments were performed in a rectilinear, prismatic channel (Fig. 1(a)), 30 m long, 0.40 m wide, with an available depth of 50 cm. The longitudinal slope of the channel was fixed at 2‰. The same conditions, involving a uniform, permanent flow, were maintained during the experiments. These are summarized in Table 1.

Table 1 Hydraulic regime in the laboratory channel experiments (Wilson-Jr, 1987).

Quantity	Unit	Laboratory channel regime: I	II	III	IV	V	VI
Q	m^3 s^{-1}	0.012	0.015	0.020	0.025	0.025	0.033
h	M	0.067	0.082	0.105	0.125	0.155	0.150
R	M	0.050	0.058	0.069	0.077	0.087	0.086
U	m s^{-1}	0.448	0.457	0.476	0.500	0.403	0.550
Re		83300	98500	121500	142400	125000	174600
Fr		0.638	0.605	0.579	0.576	0.435	0.600
τ	N m^{-2}	1.162	1.414	1.792	2.106	1.713	2.460
u_*	m s^{-1}	0.0338	0.0372	0.0419	0.0455	0.0414	0.0491

Q = flow rate; h = mean water depth; R = hydraulic radius; U = mean velocity; Re = Reynolds number; Fr = Froude number; τ = bed shear stress; u_* = shear velocity.

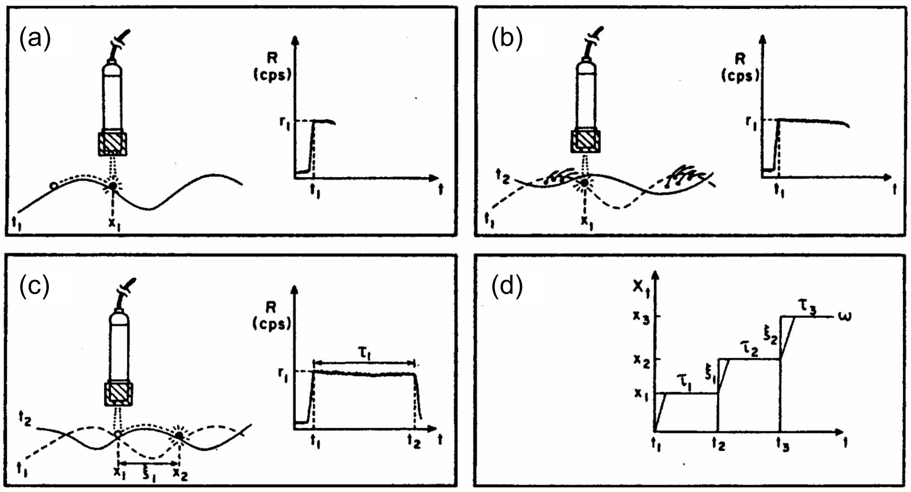


Fig. 2 Schematic diagram of step length and rest period measurements of single grains.

At the start, the mobile bed of the channel was flat, formed by a 10-cm thick layer of natural sand, with a mean diameter of 1.20 mm and a geometric standard deviation $\sigma = 1.26$. With the help of solid electrical feeders installed at the entrance of the flow, dune bed configurations were moulded and maintained in dynamic equilibrium. The bed load material was recovered in a decantation basin placed at the downstream extremity of the channel. In this manner, the bed load discharge was measured precisely.

Measurements of the movement of single grains

For the experiments with single radioactive grains, crushed glass particles with a density of 2.65 g cm^{-3} were used. The radioactive sediment grain was deposited at the bottom of the flow, in the upstream extremity. A collimated scintillation probe was deployed directly above the radioactive grain, but out of the water, during the entire experiment, as shown in Fig. 1(b). The positioning of the detector was undertaken manually, as the radioactive grain changed position. The radioactive probe was connected to a graphic recorder that continually registered the radiation from the sediment grain, as shown in Fig. 2(a). During the time the particle remained at rest at a position X_1, an almost constant radioactivity peak was registered by the recorder. If the particle remains at rest, it will be buried by other particles as the dunes move. In this case, a slight reduction in the recorded signal is observed (Fig. 2(b)). After some time, when the dune has moved, the radioactive grain will again be at the surface of the bed and the magnitude of the measured signal will return to the initial value. If the particle also moves, the precise instant of displacement, t_2, will be characterized by a reduction in the magnitude of the radioactive peak registered (Fig. 2(c)). In this manner, the rest period τ_1 will be precisely determined. Equally, with the use of the manual support that moves along the channel margin (Fig. 1(b)), the new position X_2 of the radioactive grain will be rapidly determined, at the most, 15 s later. The precision of the measurement of the grain's position is of 1–2 cm, due to the probe's lead collimator and the specific radioactivity of the grain. The distance between two successive positions of the probe represents the distance ξ_1 of a grain displacement. The longitudinal trajectories of the individual grains that constitute the individual events or the stochastic process realizations ω describing the bed load movement are therefore determined (Fig. 2(d)).

Movement of particle groups in laboratory channels

The radioactive samples were introduced with the channel stopped and without water, after it had operated for a while in hydrodynamic and sedimentological equilibrium. The injection involved the substitution of a transverse strip of sand, with a constant depth of less than 0.5 cm, by the radioactive sand containing ^{198}Au (Fig. 1(c)). In order to detect the position of the grains on the bottom, the probes were collimated so that they could register, at each instant, a strip about 5 cm length and of width equal to that of the channel. They were installed in an electric cart that moves along the channel at a constant velocity of 6.0 m min^{-1}. The transport and the longitudinal dispersal of the radioactive particles were studied with respect to time.

Movement of particle groups in natural watercourses

For this study, the selected experiments were undertaken in Horácio Creek, located in the northwest region of Paraná State, Brazil, with riverbed sand samples labelled with ^{198}Au or simulated with crushed glass containing ^{192}Ir. A 50-m long representative stretch of the stream, presented in Fig. 3(a), was prepared for the simultaneous use of classical and tracer measurements. The equipment for injecting the radioactive tracer, illustrated in Fig. 3(a)–(c), was specially designed for studies in small streams. With this equipment, the radioactive sediment was uniformly deposited on the stream bed, through the transverse injection section.

The horizontal positions of the radioactive sediment grains on the channel bed were determined with the help of three scintillation detectors, such that they could cover, at each point in time, an area of the channel bottom 5 cm long and 1 m wide, as presented in Fig. 3(a), (d) and (e). They were fixed to a metallic support that moved longitudinally along the stream on wooden rails, with a constant velocity of 2.40 m min^{-1}. Each time the probes passed along the stretch, as shown in Fig. 4(a) and (b), curves with same count rates or radioactive sediment concentrations, and three-dimensional cloud representations were produced, and the corresponding transport diagrams that represent the projections of the count rates on the longitudinal flow axis were derived (Fig. 4(c)).

MATHEMATICAL DEVELOPMENT

Considering that the particle initiated its trajectory with a rest period, that is, $x = 0$ at $t = 0$, the probability that the particle did not exceed the distance x, in the time interval $[0,t]$ is given by (Yang, 1968; Wilson-Jr, 1972, 1987) as:

$$F_t(x) = P[N(t)=0] + \sum_{n=1}^{\infty} \int_0^x [f_X(x)]^{n*} \mathrm{d}x \int_0^t \int_\tau^\infty [f_T(t)]^{n*} f_T(\tau) \mathrm{d}\tau \mathrm{d}t \tag{1}$$

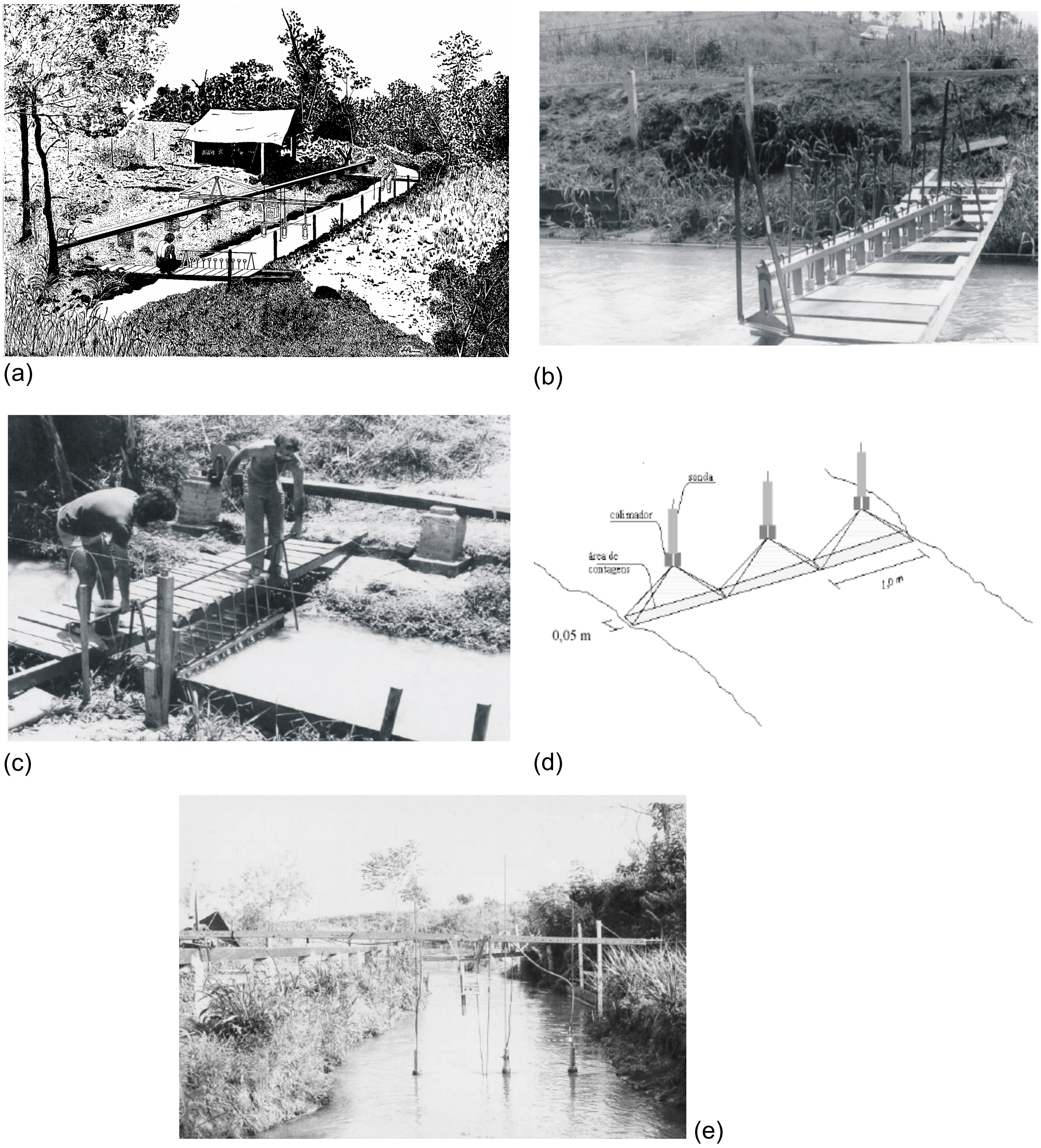


Fig. 3 (a) Measurement stretch of the Horácio Creek. (b) Radioactive sediment injection equipment. (c) Radioactive sediments injection in Horácio Creek. (d) Areas covered by the radiation detectors. (e) Detection system used in the Horácio Creek.

where:

$$P[N(t)=0]=P[T_1>t] \tag{2}$$

represents the probability that the particle did not move from its initial position in the interval $[0,t]$, that is, that the first rest period T_1 was larger than t.

$[f_X(x)]^{n*}$ represents the nth order convolution of the probability density function of the distance travelled by the particle during a displacement. It is equal to the probability density function of the distance travelled by the particle after n displacements.

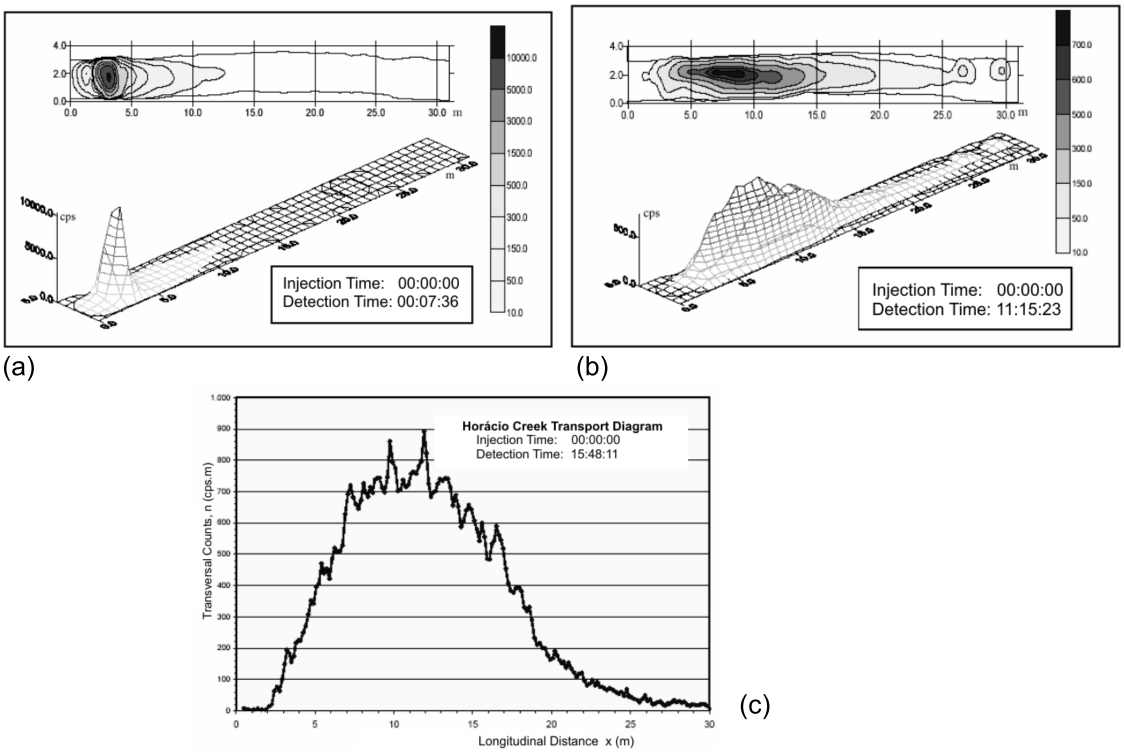


Fig. 4 Isocounts curves and 3-D representation. First experiment with ^{198}Au: (a) $t = 0.13$ h; (b) $t = 11.26$ h; (c) example of transport diagram of the Horácio Creek, $t = 15.80$ h.

$[f_T(t)]^{n*}$ is the nth order convolution of the probability density function of the duration of one rest period. It is equal to the probability density function of the duration of n successive rest periods.

The probability density function $f_t(x)$ that the particle or a group of particles will be located in the position interval $[x, x + \mathrm{d}x]$ at instant t, is given by:

$$f_t(x)=\frac{\partial}{\partial x}F_t(x)=\sum_{n=1}^{\infty}[f_X(x)]^{n*}\int_0^t\left\{[f_T(t)]^{n*}-[f_T(t)]^{(n-1)*}\right\}\mathrm{d}t \tag{3}$$

Equations (1)–(3) are general equations for stochastic modelling of bed load movement. They will generate particular models, depending on the equations obtained from the experimental data for the probability density functions $f_X(x)$ and $f_T(t)$. For instance, if the step lengths and the rest period durations were distributed according to the gamma probability laws, with mean step length of $X_m = r_2/\lambda_2$ and mean rest period of $T_m = r_1/\lambda_1$, the one-dimensional gamma function model, 1D-GM will be obtained:

$$f_t(x)=\sum_{n=1}^{\infty}\frac{\lambda_2}{\Gamma(nr_2)}(\lambda_2 x)^{nr_2-1}\mathrm{e}^{-\lambda_2 x}\int_0^t\left[\frac{\lambda_1}{\Gamma(nr_1)}(\lambda_1 t)^{nr_1-1}\mathrm{e}^{-\lambda_1 t}-\frac{\lambda_1}{\Gamma(nr_1+r_1)}(\lambda_1 t)^{nr_1+r_1-1}\mathrm{e}^{-\lambda_1 t}\right]\mathrm{d}t \tag{4}$$

When the rest periods are distributed according to an exponential probability law, $r_1 = 1$ and $T_m = 1/\lambda_1$, the model proposed by Yang (1968) will be obtained:

$$f_t(x) = \lambda_2 \, e^{-\lambda_1 t - \lambda_2 x} \sum_{n=1}^{\infty} \frac{(\lambda_1 t)^n}{\Gamma(n+1)} \frac{(\lambda_2 x)^{n r_2 - 1}}{\Gamma(n r_2)} \tag{5}$$

If, this time, the step lengths are also exponentially distributed ($r_1 = r_2 = 1$, $T_m = 1/\lambda_1$ and $X_m = 1/\lambda_2$), the Homogeneous Poisson Model studied by Yang (1968), Grigg (1970), Wilson-Jr (1972, 1987) and Wilson-Jr & Monteiro (2004) will be obtained:

$$f_t(x) = \lambda_2 \, e^{-\lambda_1 t - \lambda_2 x} \sum_{n=1}^{\infty} \frac{(\lambda_1 t)^n}{\Gamma(n+1)} \frac{(\lambda_2 x)^{n-1}}{\Gamma(n)} \tag{6}$$

In these equations, x is the longitudinal distance, t is time; λ_1 and λ_2 are temporal and spatial mobility functions of the grain; r_1 and r_2 are gamma distribution parameters; n is the number of stages; X_m is the mean displacement length; and T_m is the mean rest period duration.

DATA ANALYSIS

Movement of single particles

The one-dimensional stochastic models described by equations (4)–(6), assume that the lengths of the steps and the rest periods are distributed according to gamma probability or exponential laws whose general equations are as follows:

Gamma function:

$$\begin{aligned} f_Z(z) &= \frac{\lambda^r}{\Gamma(r)} z^{r-1} e^{-\lambda z} && \text{for} \quad z \geq 0 \\ f_Z(z) &= 0 && \text{for} \quad z < 0 \end{aligned} \tag{7}$$

Exponential function:

$$\begin{aligned} f_Z(z) &= \lambda \, e^{-\lambda z} && \text{for} \quad z \geq 0 \\ f_Z(z) &= 0 && \text{for} \quad z < 0 \end{aligned} \tag{8}$$

The r and λ parameters (r_1 and λ_1 for the rest period, and r_2 and λ_2 for the step length) were estimated from the mean and variance values of the sample distributions (Wilson-Jr, 1972, 1987). The examples presented in Figs 5 and 6 show that the gamma probability density functions adjusted perfectly to the experimental distributions of the step lengths and of the rest periods. Furthermore, due to the precise system used to detect the isolated particles that allowed the exclusive determination of rest periods smaller than 1 min, and of step lengths of 1 cm, the gamma function provides a better description of the experimental data than the exponential function.

Movement of groups of particles

The experimental confirmation that the step lengths and the rest periods are better described by gamma functions than by exponential functions indicates that the mobility of the injected particles in time and space is not constant. In reality, they vary during the

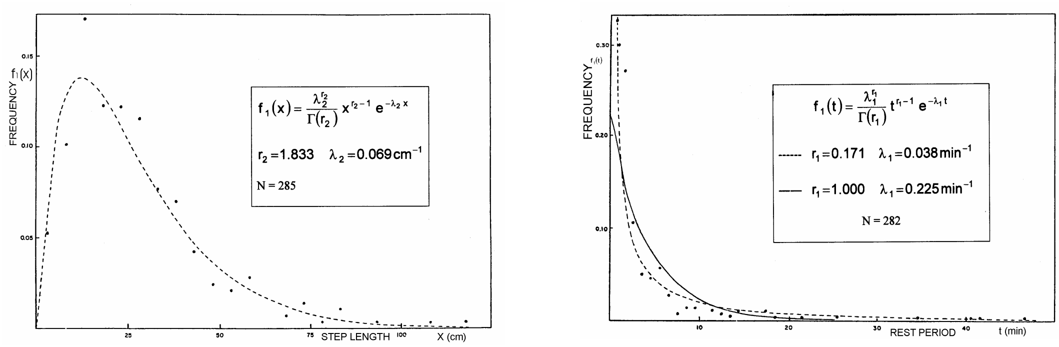


Fig. 5 Step length and rest period frequencies for a liquid discharge of 12 l s^{-1}.

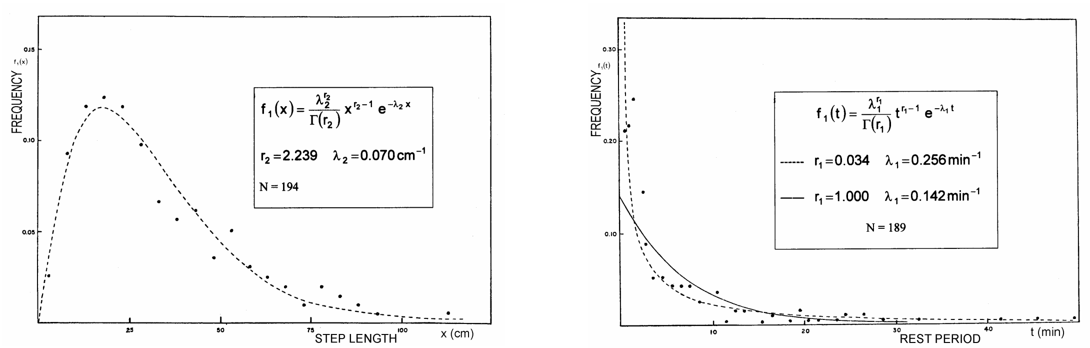


Fig. 6 Step length and rest period frequencies for a liquid discharge of 20 l s^{-1}.

time and space need for their integration in the bed mobile layer (Wilson-Jr, 1987). In plane beds, for instance, their mobility λ_1 and λ_2 initially depends on the time and distance travelled until integration. They then assume constant values. Initially, they should be described by non-homogeneous Poissonian processes of the 1D-GM type, or non-Poissonian processes, and when integrated to the bed, by homogeneous Poissonian processes of the HDPM (Homogeneous One-dimensional Poissonian Model) type (Wilson-Jr, 1987). The larger the difference in the characteristics of the injected particles, the greater the time and distance needed for their integration.

Figure 7 presents particular solutions to equation (4), for integer values of r_1. In this example, when $r_1 = 1$, the solution to Yang's One-dimensional Gamma-Exponential Model is obtained. For $r_1 = r_2 = 1$, the solution to the HDPM would be obtained.

Software named PAICON (Processos Aleatórios com Injeções Instantânea e CONtínua—Stochastic Processes with Instantaneous and Continuous Injection), was developed in DELPHI, for analysis of the movement of sediment and pollutants in open-channel flows. Figures 8 and 9 present examples of fitting the longitudinal position density probability curves to experimental results from the laboratory channel and from Horácio Creek.

CONCLUSIONS

The experimental results for single grains showed that the 1D-GM provides a better analytical explanation of the movement of bed load grains than the HDPM models, which represent only a particular solution of the 1D-GM.

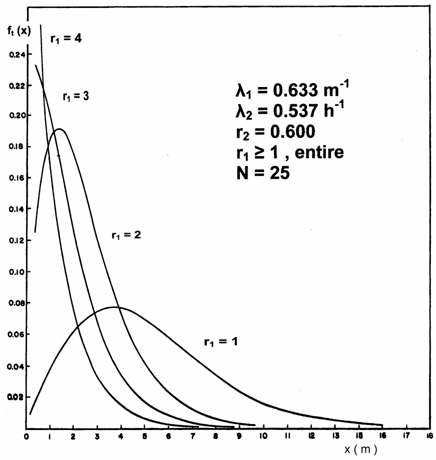


Fig. 7 Density probability functions issues of the 1D-GM, t = 15.62 h.

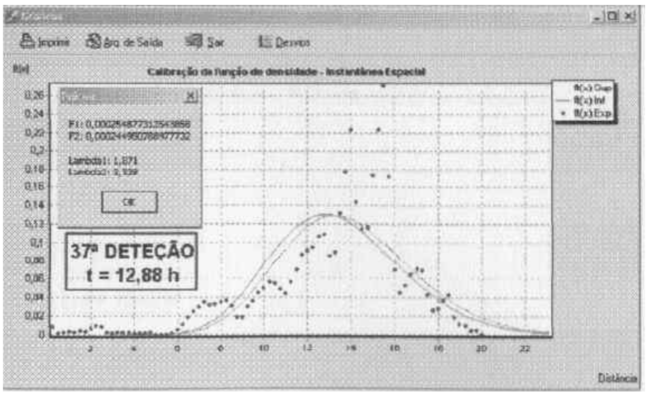


Fig. 8 Longitudinal positions of the radioactive sediments density probability curves in the laboratory channel of the UFRGS. Use of the HDPM.

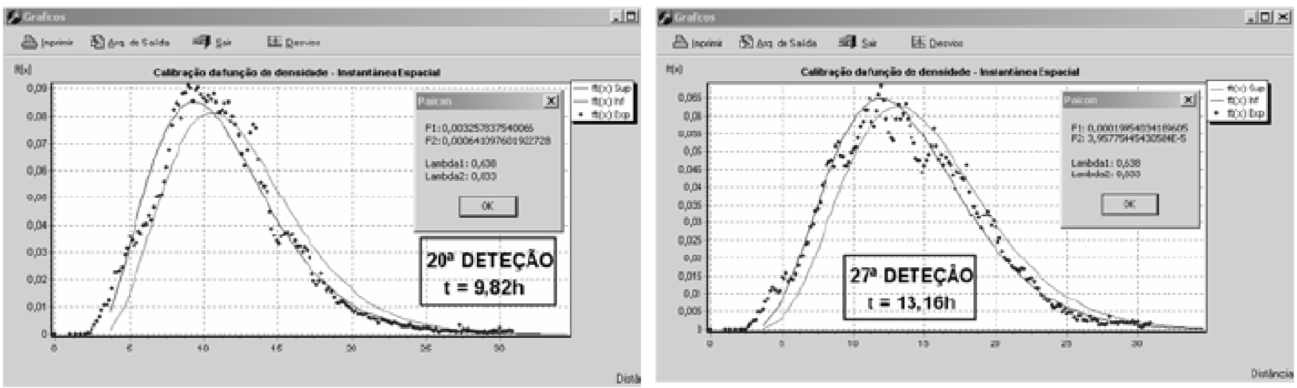


Fig. 9 Probability density curves of the radioactive sediments positions in the Horácio Creek. Campaign with ^{198}Au. (Wilson-Jr, 2004).

The first applications of the 1D-GM indicate that it can describe the behaviour of the injected sediment grains during their integration period and distance, for the plane bed case. When the injected particles possess characteristics similar to those of the bed, integration occurs rapidly and over a short distance, making the mobility functions tend to constant values.

As shown by the HDPM presented in Fig. 8, the 1D-GM, has apparently not well represented the bed load movement in the presence of regular sandy dunes, of D_{50} = 1.2 mm. However, the HDPM better represents the Horácio Creek bed load movement, which consists of ripples and dunes formed by finer sediments, with D_{50} = 0.31 mm.

Acknowledgements The support for the work reported provided by CNPq (Brazilian Research and Development Council) and COPPE/UFRJ, is gratefully acknowledged.

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